Rotational Inertia

Unit: Rotational Dynamics

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

AP Physics 1 Learning Objectives: N/A, but needed for torque and angular momentum

Knowledge/Understanding Goals:

- rotational inertia

Skills:

- calculate the rotational inertia of a system that includes one or more masses at different radii from the center of rotation

Language Objectives:

- Understand and correctly use the terms "rotational inertia" and "torque."
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Bicycle wheel.
- Gyroscope.

Notes:

<u>inertia</u>: the tendency for an object to continue to do what it is doing (remain at rest or remain in motion).

<u>rotational inertia</u> (or angular inertia): the tendency for a rotating object to continue rotating.

<u>moment of inertia</u> (I): a quantitative measure of the rotational inertia of an object. Moment of inertia is measured in units of $kg \cdot m^2$.

Inertia in linear systems is a fairly easy concept to understand. The more mass an object has, the more it tends to remain at rest or in motion, and the more force is required to change its motion. *I.e.,* in a linear system, inertia depends only on mass.

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Rotational Inertia

Rotational inertia is somewhat more complicated than inertia in a non-rotating system. Suppose we have a mass that is being rotated at the end of a string. (Let's imagine that we're doing this in space, so we can neglect the effects of gravity.) The mass's inertia keeps it moving around in a circle at the same speed. If you suddenly shorten the string, the mass continues moving at the *same speed through the air*, but because the radius is shorter, the mass makes more revolutions around the circle in a given amount of time.

In other words, the object has the same linear speed (*not* the same *velocity* because its *direction* is constantly changing), but its angular velocity (degrees per second) has increased.

This must mean that an object's moment of inertia (its tendency to continue moving at a constant angular velocity) must depend on its distance from the center of rotation as well as its mass.

The formula for moment of inertia is:

$$I = \sum_i m_i r_i^2$$

I.e., for each object or component (designated by a subscript), first multiply mr^2 for the object and then add up the rotational inertias for each of the objects to get the total.

For a point mass (a simplification that assumes that the entire mass exists at a single point):
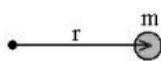
$$I = mr^2$$

This means the rotational inertia of the point-mass is the same as the rotational inertia of the object.
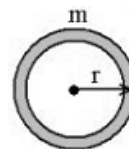
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Calculating the moment of inertia for an arbitrary shape requires calculus. However, for solid, regular objects with well-defined shapes, their moments of inertia can be reduced to simple formulas:
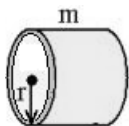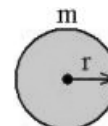
Point Mass at a Distance:
$$I = mr^2$$

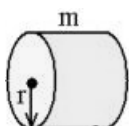Hollow Sphere:
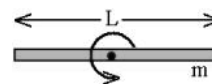$$I = \frac{2}{3}mr^2$$

Hollow Cylinder:
$$I = mr^2$$

Solid Sphere:
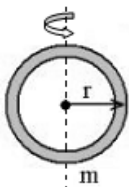$$I = \frac{2}{5}mr^2$$

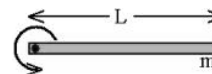Solid Cylinder:
$$I = \frac{1}{2}mr^2$$

Rod about the Middle:
$$I = \frac{1}{12}mL^2$$

Hoop about Diameter:
$$I = \frac{1}{2}mr^2$$

Rod about the End:
$$I = \frac{1}{3}mL^2$$

In the above table, note that a rod can have a cross-section of any shape; for example a door hanging from its hinges is considered a rod rotated about the end for the purpose of determining its moment of inertia.

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Sample Problem:

Q: A solid brass cylinder has a density of $8500\frac{kg}{m^3}$, a radius of 0.10 m and a
height of 0.20 m and is rotated about its center. What is its moment of
inertia?

A: In order to find the mass of the cylinder, we need to use the volume and the
density.

$$V = \pi r^2 h = (3.14)(0.1)^2 (0.2)$$

$$V = 0.00628 \, m^3$$

$$\rho = \frac{m}{V}$$

$$8\,500 = \frac{m}{0.006\,28}$$

$$m = 53.4 \, kg$$
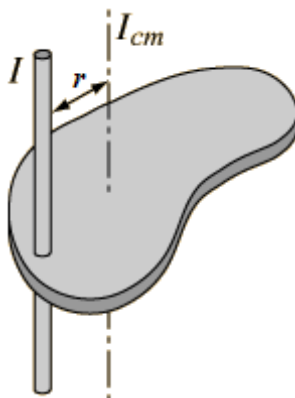
Moment of inertia of a cylinder:

$$I = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}(53.4)(0.1)^2 = 0.534 \, kg \cdot m^2$$

Parallel Axis Theorem

The moment of inertia of any object about an axis through its center of mass is
always the minimum moment of inertia for any axis in that direction in space.

The moment of inertia about any axis that is parallel to the axis through the
center of mass is given by:

$$I_{\text{parallel axis}} = I_{\text{cm}} + mr^2$$

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Note that the formula for the moment of inertia of a point mass at a distance r from the center of rotation comes from the parallel axis theorem. The radius of the point mass itself is zero, which means:

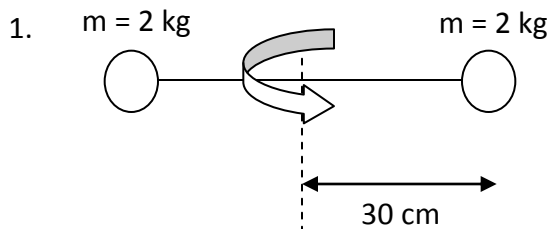$$I_{cm} = 0$$
$$I = I_{cm} + mr^2$$
$$I = 0 + mr^2$$

Note that the parallel axis theorem is beyond the scope of AP Physics 1; questions involving the parallel axis theorem will not appear on the AP Exam.
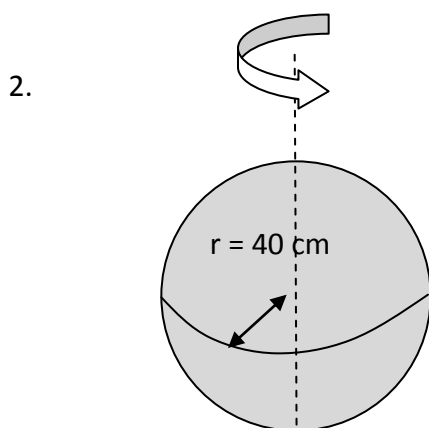
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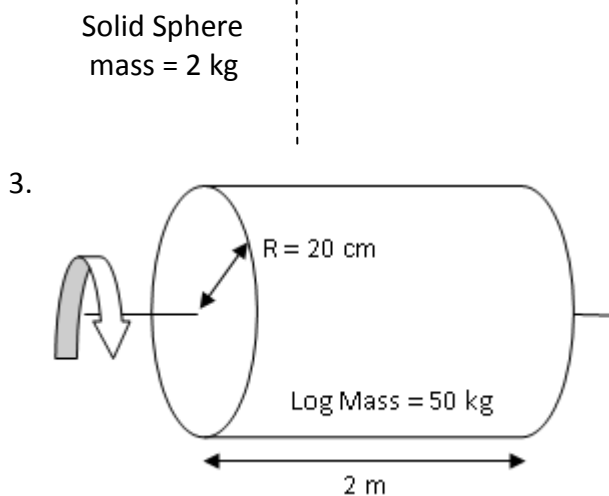
Homework Problems

Find the moment of inertia of each of the following objects. (Note that you will need to convert distances to meters.)

1.   m = 2 kg   m = 2 kg   Answer: $0.36 \, \text{kg} \cdot \text{m}^2$
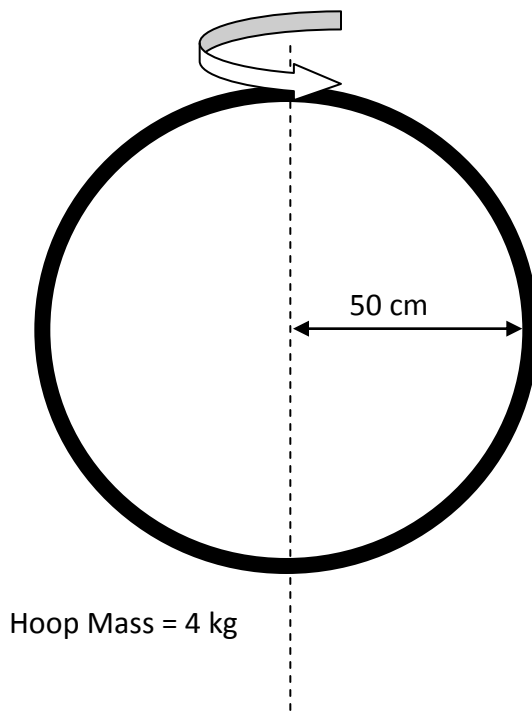
          30 cm

2.   Answer: $0.128 \, \text{kg} \cdot \text{m}^2$

      r = 40 cm

     Solid Sphere
     mass = 2 kg

3.   Answer: $1 \, \text{kg} \cdot \text{m}^2$

        R = 20 cm

                Log Mass = 50 kg

            2 m

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4.

Answer: $0.5\,\text{kg}\cdot\text{m}^2$

50 cm

Hoop Mass = 4 kg

5.

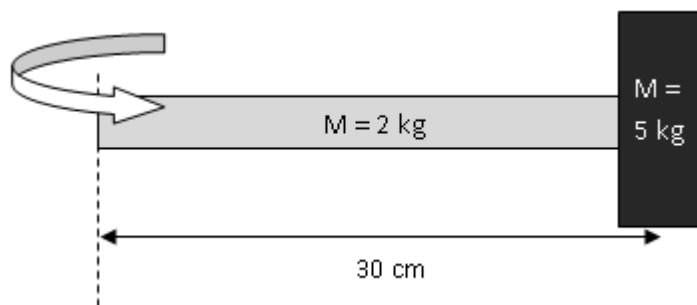Answer: $1.28\,\text{kg}\cdot\text{m}^2$

m = 6 kg

80 cm
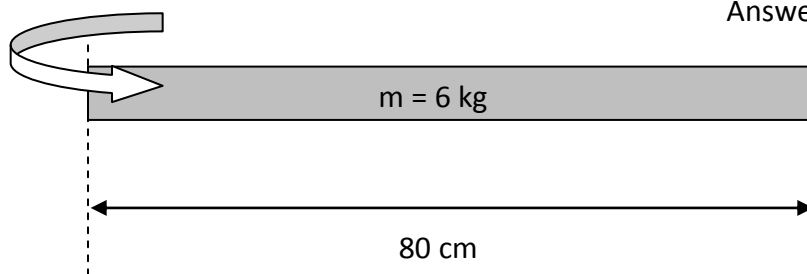
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Find the moment of inertia of each of the following compound objects. (Be careful to note when the diagram gives a diameter instead of a radius.)
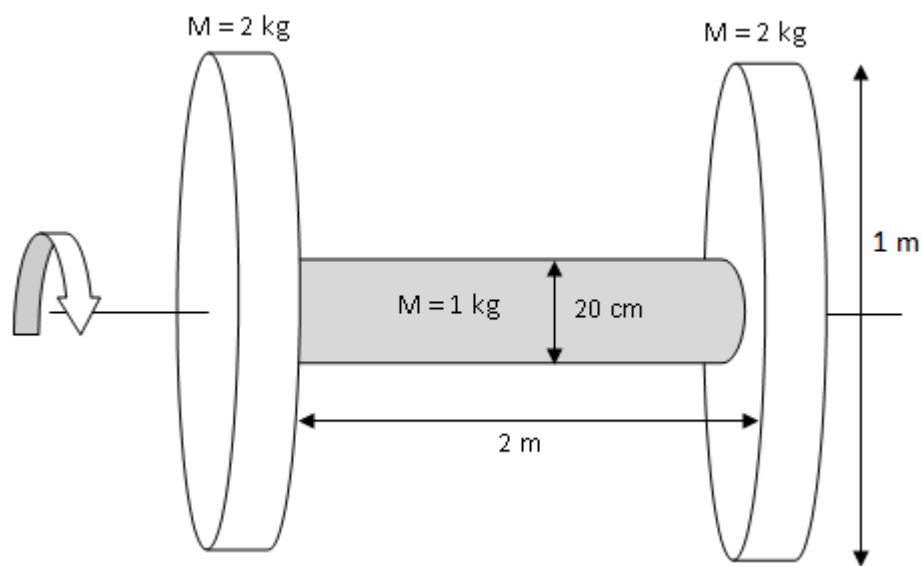
6. Sledge Hammer: Answer: $0.51\,\text{kg}\cdot\text{m}^2$

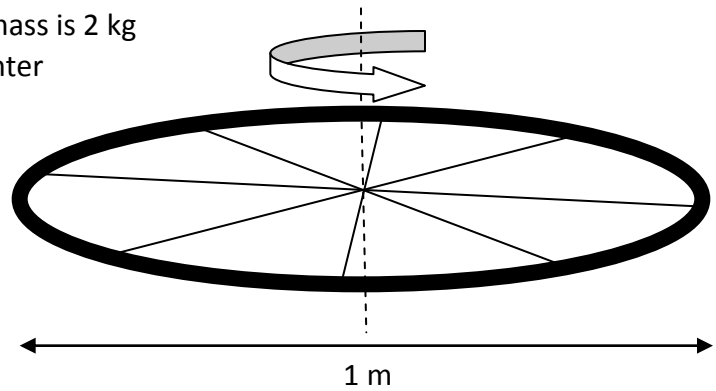7. Wheels and Axle: Answer: $0.505\,\text{kg}\cdot\text{m}^2$

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8. Wheel:

Answer: $0.8\bar{3}\,\text{kg}\cdot\text{m}^2$

- Rim (outside hoop) mass is 2 kg
- Each spoke (from center to rim) has a mass of 0.5 kg

1 m

Use this space for summary and/or additional notes.